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Light deflection by gravitational waves from localized sources

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Abstract

We study the deflection of light (and the redshift, or integrated time delay) caused by the time-dependent gravitational field generated by a localized material source lying close to the line of sight. Our calculation explicitly takes into account the full, near-zone, plus intermediate-zone, plus wave-zone, retarded gravitational field. Contrary to several recent claims in the literature, we find that the deflections due to both the wave-zone $1/r$ gravitational wave and the intermediate-zone $1/r^2$ retarded fields vanish exactly. The leading total *time-dependent* deflection caused by a localized material source, such as a binary system, is proven to be given by the quasi-static, near-zone quadrupolar piece of the gravitational field, and therefore to fall off as the inverse cube of the impact parameter.

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I. INTRODUCTION

The subject of light deflection, and/or light amplification, by gravitational waves has a long but somewhat confusing history. Early work [1,2,3] correctly concluded to the absence of first-order effects increasing linearly with the distance traversed by light within gravitational waves. This result has recently been confirmed [4], and casts doubt on the possibility of detecting or constraining a stochastic gravitational wave background by astronomical measurements (be they astrometric or photometric). However, the works supporting this pessimistic conclusion consider only purely transverse, source-free, gravitational waves and often use an indirect formalism (propagation equation for the local expansion rate of a light beam), so that their impact on the problem of light deflection by gravitational waves from localized sources is unclear. Recently, several authors have considered the case where light rays pass close to a binary gravitational-wave source and have suggested that, in such a configuration, there could arise photometric [5] or astrometric [6,7] effects proportional to the gravitational-wave amplitude $h(b)$ evaluated at the impact parameter b. If such effects $\propto h(b)$, decreasing only as $1/b$ in the wave zone of the source, existed they might fall in the detectability range of forthcoming optical interferometric arrays.

The purpose of the present paper is to study in detail the deflection of light passing near any localized, non necessarily periodic, gravitational-wave source (such as an inspiralling binary). We focus, in particular, on the effect of the time-dependent gravitational field associated with a varying quadrupole moment. As far as we know, our treatment is the first one in the literature to work out the complete effect of a time-dependent, retarded, quadrupolar field, $h(t,r)$, which is given by a sum of terms having different fall-off properties away from the source: $h(t,r) \sim a_1(t-r)r^{-1} + a_2(t-r)r^{-2} + a_3(t-r)r^{-3}$. [To save writing we suppress here indices, though our calculations take into account the full tensorial structure $h_{\mu\nu}(t,\mathbf{x})$ of the gravitational field.] We find that the claims of Refs. [5,6,7] are incorrect in that the deflection is not of order $\alpha \sim h(b) \sim a_1 b^{-1} + O(b^{-2})$, but falls off like b^{-3} : $\alpha \sim a_3 b^{-3}$. In other words, the contributions to the deflection α of both the purely wavelike field $\propto a_1(t-r)/r$ and the faster falling piece $a_2(t-r)/r^2$ cancel out to leave only the contribution of the (time-dependent) near-zone gravitational field $a_3(t-r)/r^3$. The resulting time-dependent deflection (which must be superposed on the static effect of the total mass of the source) is much too small (for reasonable impact parameters) to be of observational interest. The same pessimistic conclusion applies to the other time-dependent effects linked to $h(t,r)$: scintillation, variable redshifts and variable time delays.

II. LIGHT DEFLECTION BY A GENERIC, TIME-DEPENDENT LOCALIZED GRAVITATIONAL SOURCE

We work in the geometrical optics approximation. Let $\ell^\mu = dz^\mu/d\xi$, $\mu = 0,1,2,3$, denote the tangent 4-vector to a light ray $z^\mu(\xi)$ propagating in a curved spacetime $g_{\mu\nu}(x^\lambda)$ (with signature $-+++$). Here, ξ denotes an affine parameter along the light ray. The tangent vector ℓ^μ is (by definition) "light-like" in the technical sense of

$$\ell^2 \equiv g_{\mu\nu}(z)\ell^\mu \ell^\nu = 0\,, \tag{2.1}$$

and satisfies the geodesic equation $\ell^\lambda \nabla_\lambda \ell^\mu = 0$, or, explicitly (with $\ell_\mu \equiv g_{\mu\nu}\ell^\nu$)

$$\frac{d}{d\xi}\ell_\mu = \frac{1}{2}\ell^\alpha \ell^\beta \partial_\mu g_{\alpha\beta}(z(\xi)) \,. \tag{2.2}$$

As the main aim of the present work is to clarify the deflecting effect of locally generated bursts of gravitational waves on nearby passing light rays, we shall formally consider that both the light source and the observer are at infinity in a flat spacetime. In other words, setting $g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + h_{\mu\nu}(x)$, we neglect the effects of $h_{\mu\nu}$ near the light source and near the observer, and consider that the affine parameter ξ varies between $-\infty$ and $+\infty$. To first order in $h_{\mu\nu}$ the light deflection is given by the 4-vector

$$\Delta\ell_\mu = \ell_\mu(+\infty) - \ell_\mu(-\infty) = \frac{1}{2}\int_{-\infty}^{+\infty} d\xi \ell^\alpha \ell^\beta \partial_\mu h_{\alpha\beta}(z_0^\lambda + \xi\ell^\lambda) \,. \tag{2.3}$$

In the right-hand side of Eq. (2.3) one can consider that ℓ^α denotes the constant, incoming light-like vector $\ell^\alpha(-\infty)$, and we have replaced the photon trajectory by its unperturbed approximation $z^\lambda_{\rm unpert}(\xi) = z_0^\lambda + \xi\ell^\lambda$.

It will be technically very convenient to reexpress the deflection (2.3) in terms of the spacetime Fourier transform $\widehat{h}_{\mu\nu}(k^\lambda)$ of the gravitational field:

$$h_{\mu\nu}(x^\lambda) = \int \frac{d^4k}{(2\pi)^4}\widehat{h}_{\mu\nu}(k^\lambda)e^{ik\cdot x} \,. \tag{2.4}$$

Henceforth, we use the Minkowski metric $\eta_{\mu\nu}$ to raise and lower indices, and make use of standard flat space notations, such as $k\cdot x \equiv \eta_{\mu\nu}k^\mu x^\nu = \mathbf{k}\cdot\mathbf{x} - \omega t$, $k^2 \equiv k\cdot k$, $h \equiv \eta^{\mu\nu}h_{\mu\nu}, \ldots$ It is important to note that while ℓ^μ (the 4-momentum of the impinging photon) is on-shell, $\ell^2 = \ell\cdot\ell = 0$, the variable k^μ (4-momentum of the virtual gravitons contributing to $h_{\mu\nu}(x)$) is generically off-shell, $k^2 \neq 0$.

After inserting Eq. (2.4) into Eq. (2.3), one can perform the ξ-integration (using $\int d\xi \exp(ik\cdot z_0 + i\xi k\cdot\ell) = 2\pi\delta(k\cdot\ell)$), with the result

$$\Delta\ell_\mu = i\pi\int \frac{d^4k}{(2\pi)^4}k_\mu \ell^\alpha\ell^\beta \widehat{h}_{\alpha\beta}(k^\lambda)e^{ik\cdot z_0}\delta(k\cdot\ell) \,. \tag{2.5}$$

This result is sufficient to show that *source-free* gravitational wave packets do not deflect light. Indeed, any linearized, vacuum wave packet $h_{\alpha\beta}(x)$ is a superposition of transverse, plane waves propagating with the velocity of light. In other words, the Fourier transform $\widehat{h}_{\alpha\beta}(k)$ of a source-free wave packet contains a mass-shell delta function $\delta(k^2)$ and satisfies (independently of the coordinate gauge) the transversality condition

$$k^\alpha(\widehat{h}_{\alpha\beta}(k) - \frac{1}{2}\widehat{h}\eta_{\alpha\beta}) = 0 \qquad (\text{on shell: } k^2 = 0) \,, \tag{2.6}$$

from which follows the consequence

$$k^\alpha k^\beta \widehat{h}_{\alpha\beta}(k) = 0 \qquad (\text{on shell}) \,. \tag{2.7}$$

Coming back to Eq. (2.5), it is easy to see that when k^μ is on-shell, the delta function $\delta(k\cdot\ell)$, where *both* k^μ and ℓ^μ are on the light-cone, forces k^μ to be parallel (or antiparallel) to ℓ^μ.

The deflection is then proportional to $\ell^\alpha \ell^\beta \widehat{h}_{\alpha\beta}(k) \propto k^\alpha k^\beta \widehat{h}_{\alpha\beta}(k)$ which vanishes because of Eq. (2.7) (i.e. because of transversality). Therefore

$$\Delta\ell_\mu = 0\,, \text{ for any (localized) gravitational wave packet.} \tag{2.8}$$

This simple, algebraic argument makes it clear that the wave-like, $O(1/r)$ part of the gravitational field generated by any local source will give no contribution to the total deflection $\Delta\ell_\mu$ (when neglecting edge effects, and faster falling terms $O(1/r^2)$). We need, however, to do more work to derive the explicit, non zero value of $\Delta\ell_\mu$ generated by the contributions $O(1/r^2) + O(1/r^3) + \cdots$ to $h_{\mu\nu}(x)$. First, we need to relate $h_{\mu\nu}(x)$ to the material source, i.e. to the (localized) stress-energy tensor $T_{\mu\nu}(x)$. The linearized Einstein equations, in harmonic gauge, read

$$\Box(h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}) = -16\pi G T_{\mu\nu}\,, \tag{2.9}$$

or, in Fourier space,

$$k^2 \widehat{h}_{\mu\nu}(k) = +16\pi G(\widehat{T}_{\mu\nu}(k) - \frac{1}{2}\eta_{\mu\nu}\widehat{T}(k))\,, \tag{2.10}$$

where

$$\widehat{T}_{\mu\nu}(k) = \int d^4x e^{-ik\cdot x} T_{\mu\nu}(x)\,. \tag{2.11}$$

When dividing by k^2 to get $\widehat{h}_{\mu\nu}(k)$ from Eq. (2.10), one needs to define carefully the singularity structure at $k^2 = 0$, which is related to the boundary conditions incorporated in the corresponding Green's function. The Fourier transform of the usual *retarded* Green's function is $(k^2 - i\epsilon k^0)^{-1}$, where ϵ is a positive infinitesimal, so that

$$\widehat{h}_{\mu\nu}(k^\lambda) = 16\pi G \frac{\widehat{T}_{\mu\nu}(k) - \frac{1}{2}\eta_{\mu\nu}\widehat{T}(k)}{k^2 - i\epsilon k^0}\,. \tag{2.12}$$

Note in passing that, in the decomposition (P denoting the principal part)

$$\frac{1}{k^2 - i\epsilon k^0} = P\frac{1}{k^2} + i\pi \mathrm{sign}(k^0)\delta(k^2)\,, \tag{2.13}$$

the second (on shell) term is the only one to contribute to the "radiation" Green function $G_{\text{retarded}} - G_{\text{advanced}}$ which defines a free wave packet associated to the source $T_{\mu\nu}$ and falling off at infinity like $1/r$. By the argument above we know that this one-shell term will not contribute to $\Delta\ell_\mu$. This shows that the deflection would be the same for the physical retarded field $h^{\text{ret}}_{\alpha\beta}(x)$, or for the acausal fields $h^{\text{adv}}_{\alpha\beta}(x)$ or $h^{\text{sym}}_{\alpha\beta}(x) = \frac{1}{2}(h^{\text{ret}}_{\alpha\beta}(x) + h^{\text{adv}}_{\alpha\beta}(x))$. Let us continue working with the retarded field (2.12).

Inserting Eq. (2.12) into Eq. (2.5) we get (because of the vanishing of $\ell^\alpha \ell^\beta \eta_{\alpha\beta}$)

$$\Delta\ell_\mu = 16 i\pi^2 G \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu \ell^\alpha \ell^\beta \widehat{T}_{\alpha\beta}(k)}{k^2 - i\epsilon k^0} e^{ik\cdot z_0}\delta(k\cdot\ell)\,. \tag{2.14}$$

The energy-momentum conservation law $0 = \nabla_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} + O(hT)$, or, in Fourier space and at lowest order, $k^\nu \widehat{T}_{\mu\nu}(k) = 0$, gives explicitly

$$\widehat{T}_{0i} = -\frac{k^j}{k^0}\widehat{T}_{ij} \, , \tag{2.15}$$

$$\widehat{T}_{00} = -\frac{k^i}{k^0}\widehat{T}_{0i} = +\frac{k^i k^j}{(k^0)^2}\widehat{T}_{ij} \, , \tag{2.16}$$

so that

$$\ell^\alpha \ell^\beta \widehat{T}_{\alpha\beta}(k) = (\ell^0)^2 \left(\frac{k^i}{k^0} - \frac{\ell^i}{\ell^0}\right)\left(\frac{k^j}{k^0} - \frac{\ell^j}{\ell^0}\right)\widehat{T}_{ij}(k) \, . \tag{2.17}$$

Let us henceforth split space and time and work in the center of mass frame of the source (with the center of mass used as spatial origin). The temporal origin is fixed by the requirement that the $\xi = 0$ event on the photon worldline is spatially closest to the center of mass of the source and happens at coordinate time $t = 0$. Technically, this implies that $z_0^\lambda = (0, \mathbf{b})$ where the (vectorial) impact parameter $\mathbf{b} \equiv b^i$ is orthogonal to the photon 3-momentum $\boldsymbol{\ell} \equiv \ell^i$. We can then introduce usual polar coordinates (θ, φ) to parametrize the direction of the 3-vector $\mathbf{k}$ with respect to a spatial triad with x-axis along $\mathbf{b}$ and z-axis along $\boldsymbol{\ell}$, i.e.

$$k^\mu = (\omega, k^1, k^2, k^3) = (\omega, k\sin\theta\cos\varphi, k\sin\theta\sin\varphi, k\cos\theta) \, .$$

Henceforth, $k, \ell, \ldots$ denote the spatial lengths of the 3-vectors $\mathbf{k}, \boldsymbol{\ell}, \ldots$ and no longer 4-vectors as above. We denote also $\omega \equiv k^0$ (while $\ell^0 = |\boldsymbol{\ell}| = \ell$). From $k^\mu \ell_\mu = \mathbf{k}\cdot\boldsymbol{\ell} - \omega\ell^0 = k\ell\cos\theta - \omega\ell$ the delta function in Eq. (2.14) reads

$$\delta(k^\mu \ell_\mu) = \frac{1}{k\ell}\delta\left(\cos\theta - \frac{\omega}{k}\right) \, . \tag{2.18}$$

This implies that the integration on $\mathbf{k}$ is restricted to values such that $\mathbf{k}^2 \geq \omega^2$. When writing Eq. (2.14) explicitly, we find it convenient to replace the k-integration in $d^4k = d\omega k^2 dk d(\cos\theta) d\varphi$ by an integration over

$$u \equiv \sqrt{\mathbf{k}^2 - \omega^2} = k\sin\theta \, . \tag{2.19}$$

Finally, inserting Eqs. (2.17) and (2.18) into Eq. (2.14) we get

$$\begin{aligned}\alpha_\mu \equiv \frac{\Delta\ell_\mu}{\ell} = i\frac{G}{\pi^2}\int_{-\infty}^{+\infty}\omega^{-2}d\omega\int_0^{+\infty}udu\int_0^{2\pi}d\varphi K_\mu(\omega, u, \varphi)e^{ibu\cos\varphi} \\ [\cos^2\varphi\widehat{T}_{11}(\omega, \mathbf{k}) + \sin^2\varphi\widehat{T}_{22}(\omega, \mathbf{k}) + 2\sin\varphi\cos\varphi\widehat{T}_{12}(\omega, \mathbf{k})] \, ,\end{aligned} \tag{2.20}$$

where the denominator $(\mathbf{k}^2 - \omega^2 - i\epsilon\omega)^{-1}$ cancelled with a contribution $\propto \mathbf{k}^2 - \omega^2$ in the numerator (confirming the irrelevance of ϵ, i.e. of the Green's function boundary conditions), and where K_μ denotes the value of k_μ when restricted by the delta function (2.18), namely

$$K_\mu(\omega, u, \varphi) = (K_0, K_1, K_2, K_3) = (-\omega, u\cos\varphi, u\sin\varphi, \omega) \, . \tag{2.21}$$

Note that the result (2.20) is entirely expressed in terms of the components of $\widehat{T}_{ij}$ pertaining to the $x-y$-plane, i.e. the plane orthogonal to the direction of propagation of the incoming light. This is again an aspect of the transversality of the gravitational field.

III. LIGHT DEFLECTION BY A TIME-DEPENDENT, QUADRUPOLAR GRAVITATIONAL FIELD

To see better the physical content of the result (2.20), let us make the further approximation that the source internal motions are nonrelativistic so that the time-dependent part of the external field is well described by the quadrupolar approximation. Explicitly, this means (in x-space), a field which reads (in a suitable harmonic gauge, and after subtraction of the Schwarzschild-like, monopolar piece)

$$\begin{aligned}\widetilde{h}^{00} &= +2G\partial_{ij}\left(\frac{D_{ij}(t-r)}{r}\right) , \\ \widetilde{h}^{0i} &= -2G\partial_{j}\left(\frac{\dot{D}_{ij}(t-r)}{r}\right) , \\ \widetilde{h}^{ij} &= +2G\frac{\ddot{D}_{ij}(t-r)}{r} . \end{aligned} \tag{3.1}$$

Here $\widetilde{h}^{\mu\nu} \equiv h^{\mu\nu} - \frac{1}{2}h\eta^{\mu\nu}$ satisfies $\partial_\nu \widetilde{h}^{\mu\nu} = 0$, and

$$D_{ij}(t) = \int d^3x x^i x^j T^{00}(t, \mathbf{x}) \tag{3.2}$$

is the quadrupole moment (with its trace). Note that, when expanding the action on r of the spatial derivatives in Eqs. (3.1), the resulting retarded gravitational field contains a sum of contributions of the form $a_1(t-r)/r + a_2(t-r)/r^2 + a_3(t-r)/r^3$. The $1/r$ piece is the usual quadrupolar wave, the $1/r^3$ piece is a retarded version of the near-zone quadrupolar field $\left(\widetilde{h}^{00}_3 = 2h^{00}_3 = 2GD_{ij}(t-r)\partial_{ij}1/r\right)$ and the $1/r^2$ piece plays a role in the region intermediate between the near zone and the wave zone. Our present calculation (done below in Fourier space) takes into account all these contributions and allows one to study carefully the fall-off properties of the light-deflection $\Delta\ell_\mu$ as a function of the impact parameter.

In Fourier space, the quadrupolar approximation is easily seen (e.g. by Fourier-transforming $\Box\widetilde{h}^{ij} = -8\pi G\ddot{D}_{ij}(t)\delta^3(\mathbf{x})$) to correspond to making the approximation that

$$\widehat{T}_{ij}(\omega, \mathbf{k}) = \int d^4x e^{-i\mathbf{k}\cdot\mathbf{x}+i\omega t} T_{ij}(t, \mathbf{x}) \tag{3.3}$$

is independent of $\mathbf{k}$ [8], so that (using the standard virial theorem $\int d^3x T_{ij}(t, \mathbf{x}) = \frac{1}{2}\partial_t^2 \int d^3x x^i x^j T_{00}(t, \mathbf{x})$ following from $\partial_\nu T^{\mu\nu} = 0$)

$$\widehat{T}_{ij}(\omega, \mathbf{k}) \simeq \widehat{T}_{ij}(\omega, \mathbf{0}) = \int dt e^{i\omega t} \int d^3x T_{ij}(t, \mathbf{x}) = -\frac{\omega^2}{2} D_{ij}(\omega) , \tag{3.4}$$

where $D_{ij}(\omega) \equiv \int dt e^{i\omega t} D_{ij}(t)$.

Under this approximation, one can explicitly perform the integrations in our general result (2.20). Indeed, all the u-integrals in (2.20) are of the form

$$U_n = \int_0^\infty du u^n e^{i(b\cos\varphi)u} = n! \left(\frac{-1}{ib\cos\varphi - \epsilon}\right)^{n+1} , \tag{3.5}$$

with $n = 1$ or 2. Here, the positive infinitesimal ϵ (which is unrelated to the one entering the retarded Green's function) is mathematically justified by appealing to distribution theory, or physically justified by remembering that, in reality, $\widehat{T}_{ij}(\omega, \mathbf{k})$, Eq. (3.3), must fall off to zero as $|\mathbf{k}| \to \infty$, i.e. $u \to \infty$. Using the result (3.5), the φ-integrals in Eq. (2.20) are of the form

$$\Phi_n = \int_0^{2\pi} d\varphi \left(\frac{\sin \varphi}{\cos \varphi + i\epsilon} \right)^n , \tag{3.6}$$

with $n = 0, 1, 2$ or 3. Clearly $\Phi_0 = 2\pi$, while $\Phi_1 = \Phi_3 = 0$ by symmetry. It remains only to evaluate Φ_2 for which we find

$$\Phi_2 = -2\pi + \frac{2\pi\epsilon}{\sqrt{1+\epsilon^2}} , \tag{3.7}$$

which tends to -2π as $\epsilon \to 0$.

Finally, the two deflection angles $\alpha_1 = \Delta\ell_1/\ell$, $\alpha_2 = \Delta\ell_2/\ell$, in the plane orthogonal to the light ray (remember that the first axis is along $\mathbf{b}$, and the second is parallel to $\boldsymbol{\ell} \times \mathbf{b}$), are given by

$$\alpha_1 = -\frac{2G}{\pi b^3} \int_{-\infty}^{+\infty} d\omega [D_{11}(\omega) - D_{22}(\omega)] = -\frac{4G}{b^3} [D_{11}(t_0) - D_{22}(t_0)] , \tag{3.8}$$

$$\alpha_2 = +\frac{4G}{\pi b^3} \int_{-\infty}^{+\infty} d\omega D_{12}(\omega) = +\frac{8G}{b^3} D_{12}(t_0) . \tag{3.9}$$

Here, t_0 ($= 0$ in our coordinate system) denotes the date when the light ray passes nearest to the source. The longitudinal fractional change of the photon momentum $\alpha_3 = \Delta\ell_3/\ell$ is equal to the fractional change in photon energy $\alpha^0 = \Delta\ell^0/\ell = -\Delta\ell_0/\ell$ and is given by

$$\alpha^0 = \alpha_3 = +\frac{iG}{\pi b^2} \int_{-\infty}^{+\infty} d\omega \omega [D_{11}(\omega) - D_{22}(\omega)] = -\frac{2G}{b^2} \frac{\partial}{\partial t_0} (D_{11}(t_0) - D_{22}(t_0)) . \tag{3.10}$$

Let us note in passing that these results, here expressed in terms of the quadrupole moment (3.2) with its trace, depend only on the trace-free quadrupole moment $Q_{ij} \equiv D_{ij} - \frac{1}{3} D_{ss} \delta_{ij}$. This was a priori expected as it is well-known that, modulo a coordinate transformation, the time-dependent gravitational field external to any source depends only on $Q_{ij}(t)$. We could everywhere replace D_{ij} by Q_{ij} but we will not bother to do so.

The results (3.8)–(3.10) can be encoded in a scalar potential $V(z_0^\lambda)$ which is essentially the gravitational perturbation of the time delay between the light source and the observer. Indeed, if we define

$$V(z_0^\lambda) = \frac{1}{2\ell} \int_{-\infty}^{+\infty} d\xi \ell^\alpha \ell^\beta h_{\alpha\beta}(z_0^\lambda + \xi \ell^\lambda) , \tag{3.11}$$

we see that Eq. (2.3) yields

$$\alpha_\mu \equiv \frac{\Delta\ell_\mu}{\ell} = \frac{\partial}{\partial z_0^\mu} V(z_0^\lambda) . \tag{3.12}$$

Using the integrals given above, it is easy to obtain

$$V(z_0^\lambda) = \frac{G}{\pi b^2} \int d\omega [D_{11}(\omega) - D_{22}(\omega)] = \frac{2G}{b^2} [D_{11}(t_0) - D_{22}(t_0)] \,. \tag{3.13}$$

Then, to compute Eq. (3.12) one needs to express b, t_0, as well as the tensor projection $D_{11} - D_{22}$, as explicit functions of $z_0^\lambda = (z_0^0, z_0^i)$. This is achieved as follows: Let the system's center of mass (c.m.) worldline be denoted $y^\mu(\tau) = y_0^\mu + \tau u^\mu$, where τ is the c.m. proper time. In spacetime, the impact parameter is a 4-vector b^μ which connects $y^\mu(\tau)$ to the photon worldline $z^\mu(\xi) = z_0^\mu + \xi \ell^\mu$ and which is orthogonal to *both* worldlines: $0 = u_\mu b^\mu = \ell_\mu b^\mu$. This *bi-normal* b^μ is unique and is obtained by projecting $z^\mu(\xi) - y^\mu(\tau)$ orthogonally to the two-plane spanned by u^μ and ℓ^μ. Its origin $y^\mu(\tau_b)$ on the c.m. worldline defines the (proper) time of impact: $t_0 = \tau_b$. By working in a c.m. frame (with $u^\mu = (1,0,0,0)$ but y_0^i not necessarily zero) one easily finds

$$t_0 = \tau_b = z_0^0 - \ell^{-2} \ell^0 \ell^j (z_0^j - y_0^j) \,, \tag{3.14a}$$

$$b^0 = 0 \,, \tag{3.14b}$$

$$b^i = z_0^i - y_0^i - \ell^{-2} \ell^i \ell^j (z_0^j - y_0^j) \,, \tag{3.14c}$$

which allows one to compute the derivatives with respect to $z_0^\lambda = (z_0^0, z_0^i)$ of t_0 and $b = \sqrt{\delta_{ij} b^i b^j}$. [Note that $\partial t_0 / \partial z_0^i = -\ell^{-2} \ell^0 \ell^i = -\ell^{-1} \ell^i$ does not vanish.] As for the dependence on z_0^i of the tensor projection

$$D_{11}(t_0) - D_{22}(t_0) = 2D_{11} + D_{33} - D_{ii} = 2D_{ij} \widehat{b}^i \widehat{b}^j + D_{ij} \widehat{\ell}^i \widehat{\ell}^j - D_{ij} \delta^{ij} \,, \tag{3.15}$$

where $\widehat{b}^i \equiv b^i / b$, $\widehat{\ell}^i \equiv \ell^i / \ell$, it comes (besides the z_0^i-dependence of the time-argument t_0) from the b^i-dependence of the term $2D_{ij} \widehat{b}^i \widehat{b}^j$ in Eq. (3.15). By so computing the z_0^μ-derivative of Eq. (3.13), we verified the above direct calculation of α_μ.

Equations (3.8), (3.9), (3.10) and (3.13) are the main results of this paper.

IV. DISCUSSION AND CONCLUSIONS

Our explicit results (3.8)–(3.10) show that the suggestions Refs. [5,6,7] are incorrect because the time-dependent part of the light deflection by a localized gravitational source falls off as the *inverse cube* of the impact parameter b, instead of their suggested $\alpha \sim h(b) \propto b^{-1}$. Not only is the effect of the main $1/r$ retarded wave cancelled, but even the subleading retarded contribution $\propto 1/r^2$ has no effect. This implies that the effect of the local gravitational source will be much too small (for reasonable impact parameters, when considering chance alignments) to be of observational interest. We concentrated above on astrometric effects (geometrical deflection), but our negative conclusions apply equally well to photometric effects (scintillation) which can be directly derived from the redshift and deflection effects we computed.

Note that something rather remarkable happened in our calculations. Though we performed them for technical convenience in Fourier space, the quantity we evaluated is the line integral (2.3) in which $h_{\mu\nu}(x^\lambda)$ is the full (quadrupolar) retarded field given by Eqs. (3.1),

with the structure $h(t,r) \sim a_1(t-r)/r + a_2(t-r)/r^2 + a_3(t-r)/r^3$. The final results (3.8)–(3.10) not only depend on the fastest decaying contribution $a_3(t-r)/r^3$, but they no longer contain an integral over time. Without our making a near-zone approximation (in which one expands all the retarded quantities, $a(t-r/c) = a(t) - \frac{r}{c}\dot{a}(t) + \frac{r^2}{2c^2}\ddot{a}(t) + \cdots$) the exact results depend only on the value of the coefficient a_3 at the time t_0 of closest impact. [In particular, as we said above the results do not depend on the retarded, advanced or time-symmetric Green function used.]

In other words, our results can be stated by saying that the exact deflection in the complicated retarded field is simply obtained by computing the deflection in the t_0-instantaneous near-zone gravitational field

$$h_{00}^{\text{near zone}}(t,\mathbf{x}) = \left[\frac{2GM}{r}\right] + G\partial_{ij}\frac{D_{ij}(t)}{r} + \cdots\,, \tag{4.1}$$

$$h_{ij}^{\text{near zone}}(t,\mathbf{x}) = h_{00}^{\text{near zone}}\delta_{ij}\,. \tag{4.2}$$

Here, we have added (in brackets) to the previously considered time-dependent quadrupolar part, the static monopolar part associated to the total mass M of the gravitational source, which causes a well-known deflection $\alpha_1 = -4GM/b$. Taking advantage of this dependence on the t_0-instantaneous near-zone field, it is possible to reexpress our results (3.8)-(3.13) in a very compact (but somewhat subtle) way by considering the scalar potential (3.11) due to a unit-mass monopolar field, $h^1_{\mu\nu} = 2G\delta_{\mu\nu}/r$. One finds

$$V_1^{\text{mono}}(b) = -4G\ln b\,, \tag{4.3}$$

after discarding a formally infinite additional constant which is irrelevant in view of the later application of derivatives to $V_1^{\text{mono}}(b)$.

As the two spatial derivatives (acting on $r^{-1} = |\mathbf{x}-\mathbf{y}_0|$) in the quadrupolar term in Eq. (4.1) can be replaced by derivatives with respect to the c.m. y_0^i, we can very simply express the total, monopolar plus quadrupolar, scalar potential in terms of the unit-mass quadrupolar one

$$V^{\text{tot}}(z_0^\lambda) = MV_1^{\text{mono}}(b) + \frac{1}{2}D_{ij}(t_0)\frac{\partial^2}{\partial y_0^i\partial y_0^j}V_1^{\text{mono}}(b)\,. \tag{4.4}$$

It is easily checked (using Eqs. (3.14) to differentiate $\ln b$) that the quadrupolar piece of Eq. (4.4) yields back Eq. (3.13). Finally, using the general result (3.12), the deflection 4-vector α_μ can be written entirely as a sum of derivatives of the unit-mass monopolar potential (4.3).

In view of our present, "negative" results (absence of large enough time-dependent deflections) we did not study in as much detail the effects of the higher multipole moments. We just formally checked [by inserting in Eq. (2.20) the expansion in powers of $\mathbf{k}$ of $\widehat{T}_{ij}(\omega,\mathbf{k})$] that their contributions fall off with b at least as fast (and probably faster) than the quadrupolar one.

Let us also note in passing that our results can be easily extended to the case where gravity is not described by Einstein's theory but by a more general tensor-scalar theory.

Indeed, let us work in the "Einstein conformal frame" in which the field equations read (see, e.g., [9])

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi + 8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right), \tag{4.5}$$

$$\Box\varphi = -4\pi G\alpha(\varphi)T\,. \tag{4.6}$$

At linearized order in the deviations from a flat background $\eta_{\mu\nu}$ with constant background value of the scalar field φ_0, i.e. writing $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\varphi = \varphi_0 + \phi$, the field equations become simply (in harmonic gauge)

$$\Box h_{\mu\nu} = -16\pi G(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T)\,, \tag{4.7}$$

$$\Box\phi = -4\pi G\alpha(\varphi_0)T\,. \tag{4.8}$$

Moreover, null geodesics are conformally invariant so that light rays are null geodesics in the Einstein metric $g_{\mu\nu}$ (as well as in the Jordan-Fierz metric $\widetilde{g}_{\mu\nu} = A^2(\varphi)g_{\mu\nu}$, where the conformal factor $A(\varphi)$ is related to the $\varphi-$dependent coupling $\alpha(\varphi)$ of Eq. (4.6) by $\alpha(\varphi) = d\ln A(\varphi)/d\varphi$). The first-order geometrical deflection of a light ray is then given by the same Eq. (2.3) as above. As the first order tensor-scalar field equation (4.7) determining $h_{\mu\nu}$ is equivalent to the first-order Einstein equation (2.9) (and that $\partial_\nu T^{\mu\nu} = 0$ holds also to that order of approximation) the total deflection $\Delta\ell_\mu$ will be given by the same formula in tensor-scalar theories as in Einstein theory. Differences would appear only at the second order where the term $\partial_\mu\phi\partial_\nu\phi$ starts contributing. Note that, strictly speaking, the value and time-dependence of the quadrupole moment $D_{ij}(t)$ can differ at lowest ("Keplerian") order when considering, e.g., a binary system made of neutron stars [9]. However, this does not change the main conclusion that the deflection from localized gravitational source falls off as b^{-3}. In our present framework (where both the light source and the observer are "at infinity", i.e. in a place where the deviations from the background $(\eta_{\mu\nu}, \varphi_0)$ are neglected), there is no (time-dependent) difference at the observer between the Einstein metric, and the more "physical" Jordan-Fierz metric $\widetilde{g}_{\mu\nu}$. Therefore our result $\Delta\ell_\mu^{\rm total} \propto b^{-3}$ in the Einstein conformal frame implies the same conclusion for the physically measured deflection $\Delta\widetilde{\ell}_\mu$. The situation would be slightly more subtle if the deviations $h_{\mu\nu}$ and ϕ could not be neglected at the location of the observer (or the light source). See below our mention of edge effects in Einstein's theory. [Note that when discussing photometric effects, if ϕ cannot be neglected near the observer, one must take into account the additional area changes and variable redshifts which enter when translating Einstein-frame results into physical-frame ones.] Anyway, the main point of the present work is to discuss the importance of localized gravitational sources happening to lie close to the line of sight, and our framework is sufficient to show that these locally generated effects are much smaller than one might a priori think.

Let us also mention some useful consequences of our results. First, one could think that there remains (barring very improbable exact chance alignments) one class of physical systems where the light rays would propagate through the near zone field of a gravitational

source, namely, that where the light source is located within the gravitational source. For instance, we can think of a binary system, of which one body is emitting electromagnetic radiation. [A binary pulsar is precisely a system of this type.] Though our calculation does not really apply to such a system, it suggests very strongly that all the radiative pieces of the gravitational field (and, in particular, the slowly decreasing $1/r$ emitted retarded wave) do not contribute to the light deflection. The latter can be simply computed by using a t_0-instantaneous, static approximation to the near zone field. This confirms that the existing calculations of the local gravitational time delay (the integral of α^0) in binary pulsars [10], which used such an approximation, are accurate.

Finally, another consequence of our calculations is that, in the real case where neither the light source, nor the observer are "at infinity in a flat spacetime", our results show that the observable light deflection can be computed by neglecting localized gravitational sources, and, more generally, any quasi-localized gravitational wave packets (as we proved explicitly above, Eq. (2.8)). Therefore the observable effect will come essentially from "edge effects", i.e. from the fact that either the light source, or the observer are actually embedded in a non localized background of gravitational waves. This confirms the results of Refs. [1,2,3,4], and shows that these effects can be correctly calculated (as was done in these references) by neglecting all the source-rooted, near-zone-type parts of the total gravitational field $h_{\mu\nu}(x)$ and replacing $h_{\mu\nu}(x)$ by a pervading sea of on-shell, vacuum wave packets.

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